Proceedings of the American Academy of Arts and Sciences.

VOL. 64. NO. 3.—DECEMBER, 1929.

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Presented Oct. 9, 1929. Received Nov. 7, 1929.

CONTENTS.

Introduction.

Determinations have already been published of the effect of pressure on the rigidity of several varieties of glass.[1] The experimental method which was finally developed to make those measurements was just as capable of dealing with metals as with glass, and since the measurements on metals have a greater intrinsic interest than those on a substance of variable composition like glass, the method would have been applied in the first instance to metals, had not the complete preparations already been made for the measurements on glass, due to an erroneous preconception as to the relative magnitudes of the effect in metal and glass. In this paper the method is applied to eight metals up to 12000 kg/cm^2.

Method.

The method is essentially the same as that of the previous work. It is a differential method; two helical springs are stretched against each other, one spring consisting of the metal on which the effect is to be determined, and the other spring being of some standard metal on which the effect has already been determined by some absolute method. The standard metal was steel cut from the same spool of wire as the steel of the previous paper, and the absolute effect on it was determined by the method described in that paper, and in fact

the actual value there found was used, making no new experimental determination. The two helical springs are coupled through a short length of manganin wire, sliding over an insulated contact attached to the same frame which maintains the springs stretched. If there is a change in the relative stiffness of the two springs when hydrostatic pressure is applied, there will be a motion of the point of coupling which can be measured by determining the change in the potential drop between the contact fixed to the frame and another contact attached to the wire, a current flowing lengthwise of the manganin wire.

The experimental arrangements were practically the same as in the previous paper. The apparatus was essentially like that shown in Figure 1 of that paper, except that the sliding contact was now located in the middle of the frame, so that the two springs were of equal length, this being the disposition to give maximum sensitiveness. The dimensions of the springs were so adjusted that the stiffness of the two springs should be the same, this condition being demanded by maximum sensitiveness. The pressure part of the apparatus was the same as before; the springs were mounted in the same cylinder as before, and were rotated as before to secure freedom from friction.

The method of applying the corrections and calculating the change in rigidity from the measured changes on the potentiometer was improved and simplified as compared with that used in the previous paper. The fundamental formula for the spring was taken as before to be:

$$P = \tfrac{1}{2}\pi \frac{a^4 \mu \varphi_0^2}{s^3 \cos^2 \alpha}(l - l'),$$

where P is the stretching load, a the radius of the wire of which the spring is wound, μ the rigidity of the material of the wire, l the stretched length and l' the unstretched length of the *helical* part of the spring, s the total length of wire in the helical part, α the angle between the turns of the helix and a plane perpendicular to the axis, and φ_0 the total angle through which one end of the helix is rotated compared with the other. When there are two springs stretched against each other, P for the two springs is the same, and may be eliminated from the two equations for the two springs. Also when pressure is applied, and there is a shift of the position of the coupling point, the same equality continues to hold, the values of a, μ, l, etc. on the right hand side being now the values under pressure. The

effect of pressure on all the terms on the right hand side can be written out in terms of the geometry, known compressibilities, and the change under pressure of l given by the potentiometer measurements, except for the changes under pressure of the rigidity, which remains in the equation as unknown. If the pressure coefficient of one of the rigidities is known, as that of the steel spring by independent experiment, the other may be calculated from the equation in terms of known quantities. In the equation above, φ_0 is to be taken as not changing under pressure, the original spring being in a state of ease and so stretching without twist.

The equations actually used in the computations are:

$$(1-\chi_1 p)\left(1+\frac{d\mu_1}{\mu_1}\right)\left[1+\frac{\cos 2\alpha_1}{2\cos^2\alpha_1}\left(\frac{dl_1}{l_1}+\chi_1 p\right)\right]\left[1+\frac{dl_1+l_1'\chi_1 p}{l_1-l_1'}\right]$$

$$=(1-\chi_2 p)\left(1+\frac{d\mu_2}{\mu_2}\right)\left[1+\frac{\cos 2\alpha_2}{2\cos^2\alpha_2}\left(\frac{dl_2}{l_2}+\chi_2 p\right)\right]\left[1+\frac{dl_2+l_2'\chi_2 p}{l_2-l_2'}\right],$$

$$dl_1 = -\frac{\Delta R_0}{\rho_0}+\frac{\alpha p}{\rho_0}(R_0+\Delta R_0)+p\,[\lambda_1\chi_1+l_4\chi_3-L_0\chi_4],$$

$$dl_2 = -\,dl_1+p\,[\lambda_1\,\chi_1+\lambda_2\,\chi_2+l_3\,\chi_3-L\chi_4].$$

The meaning of most of the letters used in the formulas is obvious from figure 1. λ_1 and λ_2 are the lengths of the non-helical parts

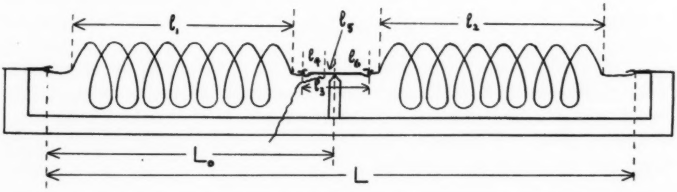

Figure 1. Scheme of the apparatus for measuring the effect of pressure on the relative rigidity of two different metals.

of the springs, that is, the lengths along the axis of the hooks on the two ends by which attachments were made. The various χ's denote the linear compressibilities of the various parts of the apparatus, taken as intrinsically positive. χ_1 is the linear compressibility of the metal to be measured, the length of the helical part of the spring of

which is l_1, χ_2 is the linear compressibility of the comparison spring of piano wire, χ_3 that of the manganin connecting wire, and χ_4 that of the supporting frame of mild steel. α is the pressure coefficient of resistance of the manganin per unit length. α is obtained from a measurement of the pressure coefficient with the terminals rigidly attached combined with the linear compressibility. In the following, $\Delta R_o/\rho_o$ will be referred to as the uncorrected value of the elongation.

The advantage of the formulas in the form given is that the stiffness of the individual springs does not enter, and no experimental determination of the amount of stretch of each spring under known weights at atmospheric pressure is necessary. The formulas of the previous paper did involve this stiffness, which was determined by direct experiment. The formulas used there were more complicated, and the stiffness appeared in two places. It now appears that the stiffness cancelled from the final result, a fact which evidently would have much simplified the previous calculations if I had noticed it in time. A second advantage of the formulas in the new form is that the measurements obviously give differential results. The difference between the pressure coefficients of the rigidities of tungsten and platinum, for example, is correct whether the value used for steel is in error or not, and if at any future time it should appear that the value used for steel was in error, the values given here can be corrected by adding the same correction to them all.

There follows now the detailed presentation of data.

Detailed Data.

In general comment on the substances chosen for investigation in the following it would appear that a number of rather uncommon metals have been included. The reason for this is in the limitations of the method. The metal has to be one which when drawn into wire and wound into a helical spring will permit a fair amount of stretch without assuming permanent set. There are not a great many metals which satisfy this requirement, and in the following I have measured all such that were readily available. The requirement of high elastic limit rules out a number of the common metals such as gold, silver, and copper.

Tantalum. This is the only metal for which more than one set of measurements was made, so that from this some idea may be obtained of the probable accuracy and significance of the results. The wire I owe to the kindness of the Research Laboratory of the General

Electric Co. at Schenectady, who supplied it to me from their regular stock. It was not very uniform in geometrical shape, the external diameter varying from 0.0223 to 0.0239 cm. Three sets of measurements were made; two of these were with the identical steel and tantalum springs, one measurement being made near the beginning of the series, and the other near the end. The purpose of the repetition was to determine the probable error due to the apparatus in measurements on the same metal. The third measurement was made on steel and tantalum springs of different dimensions from the first, and the agreement of this result with the two others will give some idea of the significance of these measurements as determining absolute constants of the metals, unaffected by accidental differences in different samples.

All the measurements of this paper on the different metals are much more irregular than measurements of most other pressure effects; in particular there is much greater tendency for a few points in each run to lie very far off the curve. There is sometimes a distinct tendency for the points to lie on one or the other of two distinct lines, the appearance being as if the springs were capable of assuming one or the other of two positions of equilibrium. The explanation of this may be found in the slight rotary motion sometimes possible to the springs due to their manner of suspension. It must be remembered that the conditions here are very much more severe than in the usual pressure experiment, the apparatus being rotated and the contact purposely broken and made several times before each reading. Perfect readings demand that the contact assume a definite unique position at each pressure, irrespective of the number of times it has been made or broken. Slight particles of dirt in the transmitting medium, which are almost impossible to avoid, produce an effect, and this is doubtless the explanation of the few points with exceptionally large discrepancies.

Of the three runs with tantalum, the first was the best. Eleven readings were made to 12000 kg/cm^2. Of these two were discarded; the remaining nine lay on a straight line with an average departure from it by a single point of 6%. The corrections were of such a magnitude as to reduce the "uncorrected" value for the shortening of the tantalum spring to a final value about one-half as large. The actual shortening under 10000 kg was 0.0034 cm. On the repetition of the experiment several months afterwards, thirteen measurements were made. Two of these had to be discarded; the remainder lay

on two straight lines of the same slope, the distance between the lines being one-third of the maximum effect. The average departure of a single reading from one or the other of these two lines was 3.3% of the maximum effect. The first set-up gave, after all corrections, an increase of rigidity of 1.49% for 10000 kg, and the repetition an increase of 1.65%. The first result is to be preferred. The run with the second set of springs gave an opposite sign for the uncorrected effect. Thirteen readings were made to 12000 kg, of which three had to be discarded, the best of them lying off the curve by 30%. The average departure from a straight line of a single one of the remaining readings was 5.4% of the maximum effect. The finally corrected result was an increase of rigidity under 10000 kg of 0.31%, about one-fifth of that found with the first springs. The determinations with the second springs are not so good as with the first, as shown by the greater irregularity of the experimental points, and the fact that the relative stiffness of the tantalum and the steel spring was not so well adjusted to give the maximum sensitiveness. Nevertheless there can be no question, I think, that all the difference between the results with the two samples cannot be ascribed to experimental error, but the two springs were doubtless actually different in their behavior under pressure. This is not so surprising when it is considered that in a drawn wire there are many internal strains, which may have been particularly great in this wire, as suggested by the lack of uniformity in the diameter.

The conclusion to be drawn from these measurements is that the rigidity of homogeneous tantalum increases under pressure by something of the order of 1% for 10000 kg.

Molybdenum. This wire I also owe to the stock of the General Electric Co. It was not perfectly round, but varied in diameter from 0.0261 to 0.0270 cm. One successful run was made to 12000 kg; several unsuccessful attempts were first made, in which there was one trouble or another with the connections and contacts. Ten readings were made in the successful run; these were best represented by a straight line, but the departures of some of the points were rather large. The average departure from a line of a single reading (no discards) was 19% of the maximum effect. The independent correction terms were nearly as large as the uncorrected elongation, but there are both positive and negative corrections, so that the final corrected elongation differs by only 2% from the uncorrected value.

The final result is an increase of rigidity of .15% under 10000 kg.

Tungsten. This was also obtained from the General Electric Co. in the form of wire 0.025 cm in diameter. Readings were made on two days with the same set-up, and three excursions were made to 12000; a number of the single pressure settings gave no readings at all, doubtless because of dirt preventing contact. Twelve readings with positive contacts were obtained; two of these were discarded, the better of them lying off a line by 40% of the maximum effect. The average departure from a straight line of the remaining 10 points was 7% of the maximum effect. The corrections, of which the largest is more than three times the uncorrected effect, combine in such a way as to make the final corrected elongation somewhat more than twice the uncorrected elongation.

The final result was decrease of rigidity of 0.29% under 10000 kg. This unexpected negative result is probably connected with the lack of perfect homogeneity of wire of this metal; when an ordinary tungsten wire is broken by repeated bending it frequently splits, in the neighborhood of the bend, into several long fibres.

Platinum. This was Baker's purest platinum, annealed, 0.0294 cm in diameter. One set-up of the apparatus was used, and readings made to 12000, back to 3000, and up to 9000 again to check at doubtful points. Eight readings in all were obtained with steady contacts. Of these two were discarded, the better lying off the line by 50%. The average departure from a straight line of a single one of the remaining 6 points was 2.6% of the maximum effect. One of the correction terms is 25% larger than the uncorrected displacement; the final corrected displacement is 11% less than the uncorrected displacement.

The final result is an increase of rigidity of 2.4% for 10000 kg.

Zirconium. This material I owe to the kindness of Dr. G. Holst of the Philips Lamp Works at Eindhoven, Holland. It is from the same length of wire as that on which I have already determined the pressure coefficient of resistance.[2] The wire was very uniform and 0.050 cm in diameter. The spring also wound much more uniformly than many of the other metals, indicating probable absence of internal strains. One set-up was used, with two excursions to 12000, making eight readings. Discarding none of these, the average departure of a single reading from a straight line was 5.5% of the maximum effect. Of these eight points, the three worst were at pressures below 4000 kg; the average departure of a single one of the

five readings above 4000 was 1.9% of the maximum effect. One of the correction terms was 66% larger than the uncorrected effect; all of the corrections combined in such a way as to make the final corrected displacement 10% less than the uncorrected displacement.

The final result was a decrease of rigidity of 0.17% for 10000 kg.

Palladium. The material was from Baker and Co., annealed wire, 0.029 cm in diameter. The results on this metal are by far the most unsatisfactory of all, and must be taken as merely of orienting value. The reason is the low elastic limit. The original spring, when stretched against the steel spring, was not far from its elastic limit, and under pressure there was an effect which amounted to an increase of set or lowering of the elastic limit. Two excursions to 12000 were made; the up and down points of the two runs lay on four straight lines like a distorted Greek sigma, the change in slope corresponding to a decrease in stiffness of the spring. After the termination of the experiment, the palladium spring was found to have received a considerable permanent elongation.

The effect of pressure on the elastic limit has apparently never been determined; what we have here is virtually a qualitative observation that the limit is lowered by pressure, an unexpected result.

Making due allowance for the effect of permanent set, the individual points of this set of measurements with palladium were perhaps better than those with any other metal, the individual points, except for one discard, lying off the straight lines by only 1 or 2%. In making the calculations, the mean of the first up and down excursion was used, as representing best the virgin wire. One of the correction terms is nearly twice the uncorrected term. The corrections combine in such a way that the final corrected displacement is of the opposite sign from and only 3% of the measured effect. That is, the corrections together almost exactly balance the measured effect.

The final result is an increase of rigidity of 1.08 % for 10000 kg.

Nickel. This was "pure" nickel from Baker and Co., 0.025 cm in diameter. A single run was made to 12000 and back, making seven readings. No points were discarded; the average departure of a single reading from a straight line was 5.7% of the maximum effect. The largest of the correction terms was 50% larger than the measured effect; the final corrected displacement was 30% less than the uncorrected displacement.

The final result is an increase of rigidity of 1.84% for 10000 kg.

Thorium. This was from the same piece of wire whose pressure coefficient of resistance I have measured,[3] and which I owe to the kindness of Dr. Rentschler of the Westinghouse Lamp Works. The section of the wire was far from regular, it was more nearly rectangular in section than either circular or elliptical, and the maximum and minimum diameters were 0.033 and 0.039 cm. Two excursions were made to 12000, obtaining 24 readings; of these 6 had to be discarded, the best departing from a straight line by 20% of the maximum effect. Of the 18 remaining readings, 12 lay on one line and 6 on another of the same slope, the average departure of a single reading from one or the other of these lines being 4.4%. The effect is unusually large, the largest correction term is only about one-half the uncorrected term, and the final corrected displacement is 54% of the uncorrected displacement and of the same sign.

The final result is an increase of rigidity of 5.73 % for 10000 kg.

Discussion.

Not a great deal of significance can be attached to the exact values found above for the pressure coefficient of the shearing modulus, as shown by the failure to obtain the same results on two springs of tantalum. One reason for this has already been suggested in the internal strains always found in any drawn wire; the importance of such strains was shown strikingly enough in the historic struggle between the multi- and the rari- constant theories of elasticity. Furthermore, the pressure coefficient of rigidity is not so definite a thing physically as the pressure coefficient of the compression modulus, for example. For a cubic crystal is equally compressed in all directions by hydrostatic pressure, and the compressibility of an aggregate is determined merely by the compressibility of the individual grains, and is independent of their relative orientations, whereas the rigidity of a microscopic aggregate of cubic crystals is connected in a more complicated way with the three elastic constants of the individual crystal grains, and will vary accordingly as the arrangement of the individual crystals is completely random or has traces of order. All of this makes it evident that the results above have a statistical rather than an individual signifflcance.

In the following table are collected the values found above for the effect of 10000 kg on the rigidity, together with the value previously found for the percentage effect of the same pressure on the incompressibility.[4] This latter is defined as $(2b/a)10^4$, where the

a and the b are the constants in the formula for the change of volume under pressure: $\Delta V/V_o = -a\,p + b\,p^2$, p being in kg/cm².

TABLE I.

SUMMARY OF THE EFFECT OF PRESSURE ON ELASTIC BEHAVIOR.

Metal.	Percentage Change of Rigidity under 10,000 kg/cm².	Percentage Change of Incompressibility under 10,000 kg/cm².
W	− .3	+ 8.8
Ta	+ 0.3 to + 1.5	+ 1.1
Mo	+ .15	+ 6.9
Zr	− .17	+ 13.6
Pt	+ 2.4	+ 10.
Th	+ 5.7	+ 28.4
Pd	+ 1.1	+ 8.1
Ni	+ 1.8	+ 7.9
Fe* (steel)	+ 2.2	+ 7.2

* Previous Result

With the exception of tungsten and zirconium, for which the effect is small, the rigidity increases under pressure. An increase is what would be expected from general considerations, and is to be contrasted with the decrease shown by several varieties of glass.

Further, the increase of rigidity under pressure is notably less than the increase of incompressibility. This again is what might be expected, and is doubtless connected with the very rapid increase in the force of repulsion between atoms when they are brought closer together than their normal distance of separation. The forces resisting volume compression are for the most part contributed by the pairs of atoms in closest contact. The repulsion between these increases rapidly when the distance between them is made less, so that the compressibility decreases by a comparatively large amount as the volume decreases under increased pressure. The forces resisting shear, on the other hand, contain a larger contribution from the more distant atoms, the component of the force effective in resisting shear contributed by the atoms directly in contact being of the second order. But the forces between distant pairs of atoms do not increase with decreasing distance as rapidly as those between adjacent pairs, and hence the effect of decrease of volume (or of

increase of pressure) must be less on rigidity than on incompressibility.

The formulas have been given in the previous paper on the rigidity of glass for the effect of pressure on Young's modulus in terms of the effect on compressibility and rigidity. It will be found in general that for the metals studied above Young's modulus increases under pressure by an amount intermediate between that of the rigidity and the incompressibility. This seems natural in the light of considerations on the nature of the atomic forces like those of the last paragraph.

I am much indebted to my assistant Mr. W. A. Zisman for making the readings and setting up the apparatus. I am also indebted to the Milton Fund of Harvard University for financial assistance.

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